Discovering Prerequisite Relationships among Knowledge Components

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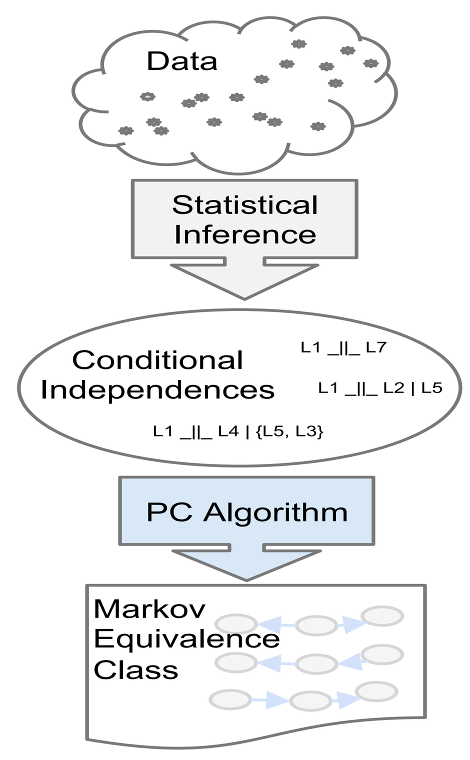
Motivation:

- Want to learn which knowledge components depend on which others.
- This "prerequisite structure" would aid curriculum design.
- No data containing natural or experimental variation in topic order.
- But we *do* have data from tests that cover multiple topics.
  Goal: use test data to learn prerequisite structure.



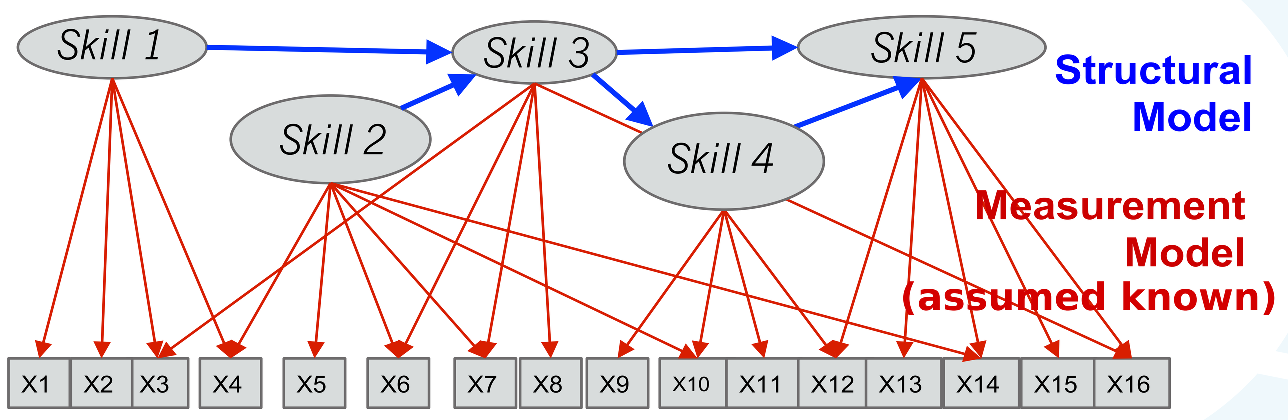
Peter Spirtes, Clark Glymour, and Richard Scheines. *Causation, Prediction and Search*. The MIT Press, Cambridge, MA., 2nd edition, 2000.

Analogy between prerequisites and causes:

- Prerequisite relationships induce conditional dependences and independences between variables in much the same way that causal relationships do.
- Machine learning algorithms exist for inferring causal relationships from conditional dependances and independences in the sample.
- We can use the same algorithms to infer prerequisite structure.

Structural search algorithm:

- PC algorithm.
- Input: a set of conditional independences over a set of variables
- Output: the Markov equivalence class of causal structures consistent with the independences.
- Structures within the equivalence class are indistinguishable without further evidence.



Measurement:

- Want to know the prerequisite relationships between Knowledge Components (KCs).
- KCs are not measured directly, so no direct measures of dependence between them.
- Need to learn dependences between latent variables to run PC.
- This is a hard problem; there is currently no general solution.

Special case: Pure measurement models

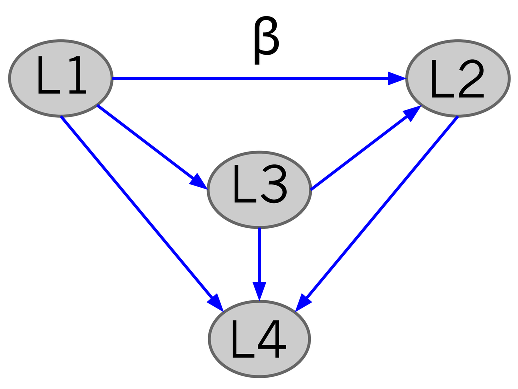
- "Pure" items load on only one latent
- When the measurement model contains many pure items, Build Pure Clusters + MIMBuild can find the structural Markov Equivalence class *without* knowing the measurement model
- However, in education, most test items load on multiple KCs
- In simulation we varied the # of pure items in the measurement model

Assumption: Measurement model is known

- **Key assumption: that we know the "Measurement Model"** (a.k.a. the Q matrix).
  I.e. for each each test item, we know which set of KCs it measures.
- Could occur by test design, or could use an existing method for Q matrix learning.

Parametric assumptions:

- Latents are continuous-valued
- Linear relationships between variables are linear
- Measured items are binary projections of underlying continuous quantities
- In simulation, we tested performance with both continuous items, and with binary projections



Method to learn independences among KCs:

- E.g.: Say we have four latents, L1-L4. Want to learn whether L1 _||_ L2 | L3.
- We can test this independence using a Structural Equation Model (SEM) fitter, by fitting the model on the left.
- It follows from d-separation that **β=0 if and only if L1 _||_ L2 | L3.**
  SEM fitting provides consistent statistical inference for parameter values.
- In fact, for any given conditional independence, we can construct a structural model such that one edge coefficient is zero if and only if that independence holds.
- We developed a construction algorithm that produces such a structural model, and proved its consistency. The details are in the full length version of our paper.

Judea Pearl. *Causality: Models, Reasoning and Inference* (2nd edition). Cambridge University Press, September 2009.

Extensions

- Sometimes multiple models test the same independence.
- Our constructor might not produce the most efficient one.
- When the Q matrix is dense or the number of KCs large, the model may be unidentifiable, so the SEM fitter will not give an answer.
- Finding the most efficient model constructor is an a potential extension.

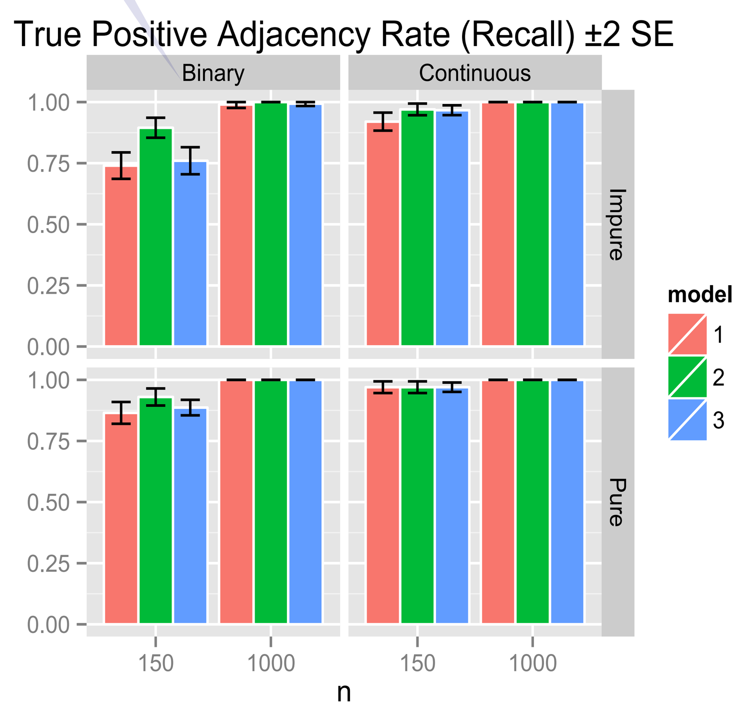
Evaluation on simulated data:

- Simulation tests how well the method recovers the Markov equivalence class when the true model is known.
- *Structure*: We tested three different structural models. The Markov equivalence classes of these models are on the right.
- *Purity*: For each structural model, we created two measurement models: one with all pure items, and one with many impure items (i.e. items that load on more than one KC). The six complete SEMs are below right.
- *Parameters*: For each complete SEM we drew 100 random parameterizations (all variables Gaussian, all relationships linear).
- *Sample size*: For each parameterization, we generated two datasets: one with 1000 simulated students' scores, and one with only 150 students' scores.
- *Binary v. Continuous items*: All these scores were continuous-valued, but real test item scores are 0-1. We projected each dataset to a binary copy, thresholding at the mean.

(a) Model 1

(b) Model 2

(c) Model 3



Results of simulation:

- The method performed well in simulation, both for edge adjacencies and edge orientations. The true positive rate for adjacencies and orientations in each condition is shown on the left.
- False positive rates in all conditions were low for both adjacencies (0·5%) and orientations (0·8%). Details are in the full paper.

Pure measurement model:

Impure model:

(a)

(b)

(c)

(d)

(e)

(f)

Conclusions:

- Our method solves a novel problem: learning the dependency structure among knowledge components from test data when the measurement model contains many impure items.
- This is a common situation in educational research.
- Applications could aid curriculum design.
- Room for improvement: extensions to the model constructor could improve efficiency & identifiability.